A risk-averse two stage stochastic programming model for transportation network protection problems

A risk-averse two-stage stochastic programming for retrofitting transportation networks

Over the past two decades, the U.S. population and vehicle ownership have both increased much faster than the total highway lane-miles. Exacerbating the situation is the limited funding for maintenance, which has resulted in even more degradation to our aging and deteriorating transportation networks. This study focuses on the problem of allocating limited retrofit resources to multiple highway bridges to improve the overall robustness of the entire transportation network. Developing such optimal retrofit program is challenging, as transportation networks are often large-scale and decision-making is subject to great uncertainty in the disasters (e.g., seismic hazards). In this study, we will integrate a risk measure such as the conditional-value-at-risk (CVaR) (Noyan, 2012)[1] into a two-stage stochastic programming framework and construct decomposition algorithms based on the benders-decomposition approach to solve the problems.

Traditional two-stage stochastic programming is risk-neutral; that is, it considers the expectations as the risk handing criterion while comparing the random variables (e.g., total cost) to identify the best decisions. However, in the presence of variability, risk measures should be incorporated into decision making process in order to model its effects. In this study, we develop a risk-averse two-stage stochastic programming model with mean-risk functions, where we specify the CVaR as the risk measure. This approach considers a variety of random outcomes to provide more robust solutions and generalizes prior modeling efforts. The models in prior studies are to minimize either the expected cost over probabilistic disaster scenarios (Liu et al., 2009)[2] using a two-stage stochastic programming, or cost in worst-case scenario (Yin et al., 2009)[3] using a robust optimization. To the best knowledge, our proposed mean-risk stochastic program is novel in the general field of transportation network protection and it captures a spectrum of risk handling criterion by simply adjusting the confidence level. In addition, we further relax assumptions adopted in previous studies and make the model accountable for considerations that are more realistic. In particular, we recommend the optimal retrofit program allowing multiple retrofit alternatives for each bridge. We also consider five states of seismic damages to a structure and allow the reductions in the probability of retrofitted bridge being damaged other zero. It is anticipated that our proposed model will be useful for policy makers at federal and state levels to establish resource allocation schemes to transportation infrastructures on a strategic level.

As the proposed model is a mixed integer nonlinear program and the integer variables appear in the second stage, the straightforward decomposition methods cannot be applied due to the non-convexity of the problem. We reformulate the problem as a convex program and then construct a decomposition method based on the general Benders Decomposition approach to solve the problem. We demonstrate the computational effectiveness of the proposed solution methods on two sample networks: a hypothetical small-scale network (e.g., a fixed charge network flow problem with reduced link capacity) and the Sioux Falls network. With the successes, we plan to implement the solution methods in real-world applications, such as the Charleston area in South Carolina.

The Mean-Risk Two-Stage Stochastic Programming Model

The model is to minimize the total cost of retrofitting the bridges and travel cost under the seismic hazards. Let denote by $G(N, A)$ a transportation network, where N is the set of nodes and A is the set of links on the network. Denote by R and S ($R \subseteq N, S \subseteq N$) the sets of origins and destinations on the network. Denote $\bar{A}$ ($\bar{A} \subset A$) with a size of $\bar{m}$ as the set of arcs that are subject to seismic hazards and thus the candidates for retrofit. The binary decision variable u_a^h is 1 if link a ($a \in \bar{A}$) is to be retrofitted

[1] Noyan, N., 2012. Risk-averse two-stage stochastic programming with an application to disaster management. *Computers & Operations Research* 39(3), 541-559.

[2] Liu, C., Fan, Y., Ordóñez, F., 2009. A two-stage stochastic programming model for transportation network protection. *Computers & Operations Research* 36(5), 1582-1590.

[3] Yin, Y., Madanat, S.M., Lu, X.-Y., 2009. Robust improvement schemes for road networks under demand uncertainty. *European Journal of Operational Research* 198(2), 470-479.

by strategy h ($h \in H$, where H denotes the set of applicable retrofit strategies) and 0 otherwise. For an origin-destination (O-D) pair (r, s), $x^{rs} \in R_+^m$ is the link flow vector, and $q^{rs} \in R_+^n$ is the vector of travel demands between each O-D pair. Denote by f_a the total flow on link a. In this study, we combine the two sets of probabilistic estimates, seismic damage to a structure and the probabilities of various earthquake occurrences, to prepare a damage prediction. Let the random vector ξ describe the uncertain events of link damages prior to any retrofit decision. Each realization and the corresponding probability $p(\xi)$ defines a damage scenario. A random vector Ξ is introduced to represent the random event of link damage in an earthquake given any retrofit strategy implemented. We assume if link $a \in \bar{A}$ is retrofitted, its probability of being damaged is reduced, and thus the remaining link capacity depends on the random event realization and retrofit strategies implemented, denoted by $g(u,\xi)$. For other links ($a \in A \setminus \bar{A}$), $g(u,\xi)$ =1. The general form of the model is presented below (1)-(7):

$$Min\,(1+\lambda)c^T u + \underset{\xi\in\Xi}{E}\{Q(u,\xi)\} + \lambda(\eta + \frac{1}{1-\alpha}\underset{\xi\in\Xi}{E}\{v(\xi)\}) \quad (1)$$

$$s.t. \quad < c,u > \le B \quad (2)$$

$$u \in \{0,1\}^{\bar{m}} \quad (3)$$

$$\text{with } Q(u,\xi) := \min_{x^{rs}} \gamma < f_a, t(f_a) > \quad (4)$$

$$W x_a^{rs} = q^{rs} \quad \forall a \in A \quad (5)$$

$$f_a = \sum_{r\in R, s\in S} x_a^{rs} \le c_a g(u,\xi) \quad \forall a \in A \quad (6)$$

$$v(\xi) \ge Q(u,\xi) - \eta \quad (7)$$

where c is the retrofit cost vector, B is the total budget for retrofitting, and c_a is the link capacity before earthquake. λ is the coefficient to link CVaR in the same objective. The link travel time t depends on the link flow f, usually described by a non-decreasing function such as the Bureau of Public Roads (BPR) function. The W represents the node-link adjacency matrix. The η is the value at risk and defined in constraint (7).

<u>*Decomposition Based Methods:*</u>

We develop a decomposition method based on the Benders-decomposition method to resolve two major computational challenges: integer variables on the nonlinear and non-convex second-stage cost function. We decompose the problem into a master problem given in (8)-(10) (feasibility cut can be neglected due to the penalty function) and one subproblem including the travel cost, which is similar to the decomposition structure discussed in (Noyan, 2012).

$$min_{u,\eta} \ (1+\lambda)c^T u + \phi_1 + \lambda\phi_2 \quad (8)$$

s.t. including (2)-(3)

optimality cut 1: $\phi_1 \ge < p(\xi), \psi(\xi) >$ (9)

optimality cut 2: $\phi_2 \ge \eta + \frac{1}{1-\alpha} < p(\xi), z(\xi) >$ (10)

where $z^\kappa(\xi)$ is the difference of second stage value $\psi(\xi)$ and the value-at risk η. We then convexify the subproblem in order to use the well-behaved algorithms, such as the primal-dual method and the subgradient method. In particular, we substitute the capacity variable on the denominator of the BPR function with a new variable and then form a new inequality constraint. At an iteration, we solve the subproblem to optimality for current first-stage decision u_a^h. Then, we generate two optimality cuts and add them to the master problem, one for the travel cost function represented by inequality (9) and the other for the CVaR term represented by inequality (10). The algorithm iterates until it reach predefined termination criterion.

We validate the solution method and evaluate its performance on the small hypothetical networks with four nodes and nine nodes. We also solve the deterministic equivalent problem (DEP) of (1)-(7) in two ways: using commercial mixed integer nonlinear program solvers such as KNITRO to directly solve or coupling branch-and-bound method with commercial nonlinear programming solvers such as CONOPT. We compare the result of the proposed solution method with the results of these two methods to justify our research efforts.